

A three-parameter neutrino mass matrix with maximal CP violation

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Abstract

Using the seesaw mechanism, we construct a model for the light-neutrino Majorana mass matrix which yields trimaximal lepton mixing together with maximal CP violation and maximal atmospheric-neutrino mixing. We demonstrate that, in our model, the light-neutrino mass matrix retains its form under the one-loop renormalization-group evolution. With our neutrino mass matrix, the absolute neutrino mass scale is a function of $|U_{e3}|$ and of the atmospheric mass-squared difference. We study the effective mass in neutrinoless $\beta\beta$ decay as a function of $|U_{e3}|$, showing that it contains a fourfold ambiguity.

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1 Introduction

The specific features of lepton mixing [1, 2] inspire the search for lepton mass matrices which might reflect some of those features in a natural way. In our previous paper [3] we have constructed a model predicting trimaximal lepton mixing. This is defined by $|U_{e2}|^2 = |U_{\mu 2}|^2 = |U_{\tau 2}|^2 = 1/3$, where U is the lepton mixing matrix. Trimaximal mixing is a less stringent requirement than tri-bimaximal mixing [4], which furthermore imposes $|U_{\mu 3}| = |U_{\tau 3}|$ and $U_{e3} = 0$. In this paper we shall further constrain our model of [3] by adding to it the non-standard CP symmetry first introduced in [5].¹

Let us list the steps in the construction of the model and, simultaneously, illustrate how we proceed to reduce the number of parameters in the lepton sector:

- Neutrinos are assumed to be of the Majorana type. The smallness of the neutrino masses is explained through the (type I) seesaw mechanism [7].
- Family symmetries—the discrete group $\Delta(27)$ together with three \mathbb{Z}_2 groups—are imposed, and afterwards broken softly and spontaneously in a sophisticated way. These symmetries justify specific forms for the charged-lepton mass matrix M_ℓ , which is diagonal, and for the neutrino Dirac mass matrix M_D , which is proportional to the unit matrix. The family symmetries also reduce the number of Yukawa couplings to a minimum and strongly constrain the Majorana mass matrix of the right-handed neutrinos M_R . Since M_ℓ and M_D are both diagonal, lepton mixing originates solely in M_R .
- We get rid of two phases by assuming invariance of the Lagrangian under the non-standard CP transformation of [5], which is eventually broken spontaneously at the electroweak scale.

The specific construction of the model will be explained in the next section; we anticipate that it produces the three-parameter light-neutrino mass matrix [3]

$$\mathcal{M}_\nu = \begin{pmatrix} x + y & z + \omega^2 y & z + \omega y \\ z + \omega^2 y & x + \omega y & z + y \\ z + \omega y & z + y & x + \omega^2 y \end{pmatrix}, \quad \text{with } \omega = e^{2\pi i/3} \text{ and } x, y, z \in \mathbb{R}, \quad (1)$$

in the basis where M_ℓ is diagonal. Our model thus predicts the neutrino masses and lepton mixing—a total of nine observables—in terms of just three parameters—the real numbers x , y , and z . The contribution of the non-standard CP transformation to the form of \mathcal{M}_ν is to constrain these numbers to be real; in our original model [3] they could be complex and, therefore, the number of physical parameters in \mathcal{M}_ν was five.

With the \mathcal{M}_ν of equation (1), trimaximal mixing is realized since

$$\mathcal{M}_\nu \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad (2)$$

¹Earlier, that symmetry had already been proposed in [6], but only for neutrinos and not within a full model.

where $\lambda = x + 2z$. Furthermore, as a consequence of the non-standard CP transformation, \mathcal{M}_ν fulfills [5]

$$\mathcal{S}\mathcal{M}_\nu\mathcal{S} = \mathcal{M}_\nu^*, \quad \text{where } \mathcal{S} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \quad (3)$$

Let us discuss the predictions of equation (1) for the neutrino masses and for lepton mixing. We follow the convention in the Review of Particle Physics [8] for the parameterization of the lepton mixing matrix. As shown in [5], it follows from equation (3) that

$$s_{23} = \frac{1}{\sqrt{2}}, \quad (4)$$

$$e^{i\delta} = \pm i, \quad (5)$$

i.e. maximal atmospheric-neutrino mixing and maximal CP violation. Moreover, there are no CP -violating Majorana phases, *i.e.* the values of those phases are trivial (0 or π). On the other hand, it follows [3] from equation (2) that

$$s_{12}^2 = \frac{1}{3(1 - s_{13}^2)}, \quad (6)$$

which relates s_{12} with s_{13} and provides the lower limit

$$s_{12}^2 \geq \frac{1}{3} \quad (7)$$

on s_{12} . The mass matrix (1) does not determine the type of neutrino mass spectrum; it could be normal—wherein the smallest neutrino mass $m_s = m_1$ —or inverted, *i.e.* $m_s = m_3$. Still, equation (1) does fix the absolute neutrino mass scale as a function of both $\Delta m_{\text{atm}}^2 = |m_3^2 - m_1^2|$ and s_{13}^2 ; indeed,

$$m_s + \sqrt{m_s^2 + \Delta m_{\text{atm}}^2} = \left[\frac{(\Delta m_{\text{atm}}^2)^2}{3s_{13}^2(2 - 3s_{13}^2)} \right]^{1/4}. \quad (8)$$

(This is a result from [3] specialized to $s_{23}^2 = 1/2$.) In summary, due to \mathcal{M}_ν containing only three parameters, there are in our model six predictions for the nine physical observables following from \mathcal{M}_ν :

- the two Majorana phases, the Dirac phase δ , and the atmospheric mixing angle are all fixed;
- the solar-neutrino mixing angle is a function of s_{13}^2 through equation (6);
- the smallest mass m_s is a function of both s_{13}^2 and Δm_{atm}^2 through equation (8).

In that sense, the parameters x , y , and z in \mathcal{M}_ν can be traded for s_{13}^2 , Δm_{atm}^2 , and $\Delta m_{\odot}^2 \equiv m_2^2 - m_1^2$.

We want to stress that the \mathcal{M}_ν of equation (1) is different from the mass matrices based on μ - τ interchange symmetry. With that symmetry one obtains $s_{13} = 0$, therefore CP violation in neutrino oscillations is absent.² In the model of this paper, on the other hand, s_{13} does not need to vanish and CP violation in neutrino oscillations is maximal. Recent fits to the oscillation data [2] indicate the possibility that $s_{13}^2 > 0$ at the low scale; this would, if confirmed, disfavour a μ - τ -symmetric \mathcal{M}_ν .

We also want to stress that the most general \mathcal{M}_ν satisfying equation (2) is

$$\mathcal{M}_\nu = \begin{pmatrix} r+s & u & t \\ u & r+t & s \\ t & s & r+u \end{pmatrix}, \quad \text{with } r, s, t, u \in \mathbb{C} \quad (9)$$

and $\lambda = r + s + t + u$. The \mathcal{M}_ν of equation (9) contains seven parameters: the moduli of r, s, t, u and their relative phases. Our light-neutrino mass matrix (1) is clearly a much restricted version of (9). Notice that, with the \mathcal{M}_ν of equation (9), tri-bimaximal lepton mixing is attained when $t = u$.

This paper is organized as follows. In section 2 we discuss the model of [3] under the constraint of the non-standard CP symmetry of [5], demonstrating that the neutrino mass matrix (1) is obtained. In section 3 we study the effect of the one-loop renormalization-group evolution on \mathcal{M}_ν and show that, in our model, the form (1) of \mathcal{M}_ν is not changed by that evolution, so that the ensuing predictions hold irrespective of the energy scale. The computation of the effective mass for neutrinoless $\beta\beta$ decay is the subject matter of section 4. We present our conclusions in section 5.

2 The model

The model that we have put forward in [3] is an extension of the Standard Model, with gauge group $SU(2) \times U(1)$. The lepton multiplets are the standard left-handed $SU(2)$ doublets $D_{\alpha L} = (\nu_{\alpha L}, \alpha_L)^T$, the right-handed charged-lepton $SU(2)$ singlets α_R , and four right-handed-neutrino $SU(2)$ singlets $\nu_{\alpha R}$, ν_{0R} ($\alpha = e, \mu, \tau$). The scalar sector of the Standard Model is also extended to four $SU(2)$ doublets ϕ_α, ϕ_0 , together with a complex gauge singlet S .

Next we discuss the symmetries of the model. The two 3×3 matrices

$$F = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix} \quad (10)$$

satisfy $F^3 = T^3 = \mathbb{1}$ and do not commute. Together they generate one of the two three-dimensional faithful irreducible representations of the discrete group $\Delta(27)$ [10]; the other one is generated by F and T^* . We display in table 1 the way in which the multiplets with index α transform under this horizontal symmetry $\Delta(27)$. The multiplets without index

²For a model with a three-parameter neutrino mass matrix based on μ - τ interchange symmetry, see [9].

	$D_{\alpha L}$	α_R	$\nu_{\alpha R}$	ϕ_α
F	F	F	F	F
T	T	T^*	T	T^*

Table 1: Transformation properties of the $\Delta(27)$ triplets under F and T .

α are T -invariant and transform as

$$\nu_{0R} \rightarrow \omega \nu_{0R}, \quad \phi_0 \rightarrow \phi_0, \quad S \rightarrow \omega S \quad \text{under } F. \quad (11)$$

Next we introduce three \mathbb{Z}_2 symmetries $\mathbf{z}_{e,\mu,\tau}$ as [11]

$$\mathbf{z}_\alpha : \quad \alpha_R \rightarrow -\alpha_R, \quad \phi_\alpha \rightarrow -\phi_\alpha, \quad (12)$$

and all other multiplets remain unchanged. Finally, we come to the non-standard CP transformation [5], defined by

$$D_{\alpha L} \rightarrow i\mathcal{S}_{\alpha\beta}\gamma_0 C \bar{D}_{\beta L}^T, \quad \alpha_R \rightarrow i\mathcal{S}_{\alpha\beta}\gamma_0 C \bar{\beta}_R^T, \quad \nu_{\alpha R} \rightarrow i\mathcal{S}_{\alpha\beta}\gamma_0 C \bar{\nu}_{\beta R}^T, \quad \nu_{0R} \rightarrow i\gamma_0 C \bar{\nu}_{0R}^T, \quad (13)$$

for the fermions (C is the charge-conjugation matrix) and

$$\phi_\alpha \rightarrow \mathcal{S}_{\alpha\beta}\phi_\beta^*, \quad \phi_0 \rightarrow \phi_0^*, \quad S \rightarrow S^* \quad (14)$$

for the scalars. The matrix \mathcal{S} is given in equation (3).

The multiplets and symmetries lead to the Yukawa couplings [3]

$$\mathcal{L}_Y = -y_1 \sum_{\alpha=e,\mu,\tau} \bar{D}_{\alpha L} \alpha_R \phi_\alpha - y_4 \sum_{\alpha=e,\mu,\tau} \bar{D}_{\alpha L} \nu_{\alpha R} (i\tau_2 \phi_0^*) + \frac{y_5}{2} \nu_{0R}^T C^{-1} \nu_{0R} S + \text{H.c.} \quad (15)$$

In the Majorana mass terms we allow soft breaking of the symmetry T , but not of F . Therefore, [3]

$$\begin{aligned} \mathcal{L}_{\text{Maj}} = & \frac{M_0}{2} \sum_{\alpha=e,\mu,\tau} \nu_{\alpha R}^T C^{-1} \nu_{\alpha R} + M_1 (\nu_{eR}^T C^{-1} \nu_{\mu R} + \nu_{\mu R}^T C^{-1} \nu_{\tau R} + \nu_{\tau R}^T C^{-1} \nu_{eR}) \\ & + \frac{M_2}{2} (\nu_{eR}^T + \omega \nu_{\mu R}^T + \omega^2 \nu_{\tau R}^T) C^{-1} \nu_{0R} + \text{H.c.} \end{aligned} \quad (16)$$

Note the consequences of the non-standard CP symmetry:

$$y_1, y_4, y_5, M_0, M_1, M_2 \in \mathbb{R}. \quad (17)$$

Since the first term in \mathcal{L}_Y has one common coupling constant y_1 , upon spontaneous symmetry breaking one needs three different vacuum expectation values (VEVs) $v_\alpha = \langle \phi_\alpha^0 \rangle_0$ to be able to account for the different charged-lepton masses:

$$m_e : m_\mu : m_\tau = |v_e| : |v_\mu| : |v_\tau|. \quad (18)$$

From equations (15) and (16), we find the Majorana and Dirac neutrino mass matrices

$$M_R = \begin{pmatrix} M_0 & M_1 & M_1 & M_2 \\ M_1 & M_0 & M_1 & \omega^2 M_2 \\ M_1 & M_1 & M_0 & \omega M_2 \\ M_2 & \omega^2 M_2 & \omega M_2 & M_N \end{pmatrix}, \quad M_D = \begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \\ 0 & 0 & 0 \end{pmatrix}, \quad (19)$$

respectively. We have defined $M_N = y_5 v_S^*$ and $a = y_4 v_0$, where v_S is the VEV of S and v_0 is the VEV of the neutral component of ϕ_0 .

With $\mathcal{M}_\nu = -M_D^T M_R^{-1} M_D$, the mass matrix (1) is obtained. Since $M_{0,1,2} \in \mathbb{R}$, it follows that the parameters x and z of \mathcal{M}_ν are real as well—see [3]. On the other hand, $y \propto 1/M_N$ [3]; therefore, the question whether y is real boils down to the question whether the minimum of the scalar potential V occurs for a real v_S . For this reason, we next investigate V .

The terms of dimension four in V must be invariant under F , T , and $\mathbf{z}_{e,\mu,\tau}$. Terms of dimension three are invariant under F and $\mathbf{z}_{e,\mu,\tau}$, but they may violate T , *cf.* equation (16). If one requires terms of dimension two to be invariant under $\mathbf{z}_{e,\mu,\tau}$ as well, then there are four $U(1)$ symmetries in the potential, one for each Higgs doublet; these symmetries are all spontaneously broken, resulting in three physical Goldstone bosons. To avoid this problem we break the \mathbf{z}_α softly by terms of dimension two. As for the symmetry F , we want it to be spontaneously broken through different VEVs $v_{e,\mu,\tau}$; in order to guarantee that this happens, we must admit F to be softly broken by terms of dimension two too. The only symmetry respected by the full scalar potential is the CP symmetry. Thus,

$$\begin{aligned} V = & \lambda_1 \left[(\phi_e^\dagger \phi_e)^2 + (\phi_\mu^\dagger \phi_\mu)^2 + (\phi_\tau^\dagger \phi_\tau)^2 \right] + \lambda_2 (\phi_0^\dagger \phi_0)^2 \\ & + \lambda_3 (\phi_e^\dagger \phi_e \phi_\mu^\dagger \phi_\mu + \phi_\mu^\dagger \phi_\mu \phi_\tau^\dagger \phi_\tau + \phi_\tau^\dagger \phi_\tau \phi_e^\dagger \phi_e) + \lambda_4 \phi_0^\dagger \phi_0 (\phi_e^\dagger \phi_e + \phi_\mu^\dagger \phi_\mu + \phi_\tau^\dagger \phi_\tau) \\ & + \lambda_5 (\phi_e^\dagger \phi_\mu \phi_\mu^\dagger \phi_e + \phi_\mu^\dagger \phi_\tau \phi_\tau^\dagger \phi_\mu + \phi_\tau^\dagger \phi_e \phi_e^\dagger \phi_\tau) + \lambda_6 \phi_0^\dagger (\phi_e \phi_e^\dagger + \phi_\mu \phi_\mu^\dagger + \phi_\tau \phi_\tau^\dagger) \phi_0 \\ & + |S|^2 \left[\lambda_7 (\phi_e^\dagger \phi_e + \phi_\mu^\dagger \phi_\mu + \phi_\tau^\dagger \phi_\tau) + \lambda_8 \phi_0^\dagger \phi_0 \right] \\ & + \lambda_9 \left[S^2 (\phi_e^\dagger \phi_e + \omega^2 \phi_\mu^\dagger \phi_\mu + \omega \phi_\tau^\dagger \phi_\tau) + S^{2*} (\phi_e^\dagger \phi_e + \omega \phi_\mu^\dagger \phi_\mu + \omega^2 \phi_\tau^\dagger \phi_\tau) \right] \\ & + \bar{\mu} \left[S (\phi_e^\dagger \phi_e + \omega \phi_\mu^\dagger \phi_\mu + \omega^2 \phi_\tau^\dagger \phi_\tau) + S^* (\phi_e^\dagger \phi_e + \omega^2 \phi_\mu^\dagger \phi_\mu + \omega \phi_\tau^\dagger \phi_\tau) \right] \\ & + \mu |S|^2 + \lambda |S|^4 + \mu_1 (S^3 + S^{*3}) + \mu_2 (S^2 + S^{*2}) + \mu_3 (S + S^*) \\ & + \sum_{i,j=0,e,\mu,\tau} (\mathcal{M}_\phi)_{ij} \phi_i^\dagger \phi_j. \end{aligned} \quad (20)$$

All the parameters in V are real; the only exception are the parameters of the 4×4 matrix \mathcal{M}_ϕ , which has the structure

$$\mathcal{M}_\phi = \begin{pmatrix} a & b & c & c^* \\ b & d & e & e^* \\ c^* & e^* & f & g \\ c & e & g^* & f \end{pmatrix}, \quad \text{with } a, b, d, f \in \mathbb{R}, \quad (21)$$

in the basis $(\phi_0, \phi_e, \phi_\mu, \phi_\tau)$. Note that c , e , and g are in general complex. These conditions derive from hermiticity together with CP invariance. Note that \mathcal{M}_ϕ is still general enough to allow for different VEVs v_i ($i = 0, e, \mu, \tau$).

Since the VEVs v_i are of the electroweak scale, whereas v_S is of the seesaw scale, a fine-tuning is necessary in V , with extremely small coupling constants λ_7 , λ_8 , λ_9 , and $\bar{\mu}$. This is an unpleasant feature of the model, which is shared, though, by all other models with two widely different mass scales.

Now we address the conditions for obtaining a real VEV of S . For this purpose we only have to consider the penultimate line of equation (20). Obviously, if

$$\text{sign } \mu_1 = \text{sign } \mu_3 \quad \text{and} \quad \mu_2 < 0, \quad (22)$$

then the absolute minimum of V is attained for real v_S ($v_S > 0$ for $\mu_1 < 0$ and $v_S < 0$ for $\mu_1 > 0$).

We find it useful to summarize the symmetry breaking in our model:

- At the seesaw scale, T is softly broken by terms of dimension three in \mathcal{L}_{Maj} , while F is spontaneously broken by the VEV of S .³
- At the electroweak scale, T , F , and the \mathbf{z}_α are all softly broken in the matrix \mathcal{M}_ϕ . All symmetries, including the non-standard CP symmetry, are broken spontaneously at this scale.

The CP symmetry is broken by $|v_\mu| \neq |v_\tau|$; the different VEVs are necessary in our model for having $m_\mu \neq m_\tau$ —see equation (18). Now we want to show that indeed there is no violation of CP for degenerate masses $m_\mu = m_\tau$. In that case we may transform

$$M_R \rightarrow K M_R K^T \quad \text{with} \quad K = \begin{pmatrix} 1 & 0_{1 \times 2} & 0 \\ 0_{2 \times 1} & K' & 0_{2 \times 1} \\ 0 & 0_{1 \times 2} & 1 \end{pmatrix}, \quad (23)$$

where K' is a 2×2 unitary matrix. Choosing

$$K' = \frac{1}{\sqrt{2}} \begin{pmatrix} i & -i \\ 1 & 1 \end{pmatrix}, \quad (24)$$

one sees that

$$M_R \rightarrow \begin{pmatrix} M_0 & 0 & \sqrt{2}M_1 & M_2 \\ 0 & M_1 - M_0 & 0 & \sqrt{3}M_2/\sqrt{2} \\ \sqrt{2}M_1 & 0 & M_1 + M_0 & -M_2/\sqrt{2} \\ M_2 & \sqrt{3}M_2/\sqrt{2} & -M_2/\sqrt{2} & M_N \end{pmatrix} \quad (25)$$

becomes real. Thus, it is indeed $|v_\mu| \neq |v_\tau|$ which breaks the CP symmetry, at the electroweak scale. Notice that the phases of the VEVs are irrelevant in the breaking of CP .

³Notice that the VEV of S is crucial in our model in order to obtain $M_N \neq 0$. If it were not for $M_N \neq 0$, the seesaw mechanism would be unable to suppress the masses of all three light neutrinos, because $\det M_R \propto M_N$.

3 The renormalization-group evolution

Throughout this section, we use indices $i, j, \dots = 0, e, \mu, \tau$ to refer to the four Higgs doublets of our model.

We use the formalism of our paper [12]. In that paper we have considered a multi-Higgs-doublet extension of the Standard Model supplemented by effective dimension-five operators

$$\mathcal{O}_{ij} = \sum_{\alpha, \beta=e, \mu, \tau} \kappa_{\alpha\beta}^{(ij)} (\nu_{\alpha L}^T \phi_i^0 - \alpha_L^T \phi_i^+) C^{-1} (\nu_{\beta L} \phi_j^0 - \beta_L \phi_j^+), \quad (26)$$

where the $\kappa_{\alpha\beta}^{(ij)}$ are coefficients with dimension -1 . This is sufficient for our purposes despite the occurrence, in our model, of a scalar singlet S . The reason is that S is integrated out at the seesaw scale m_R , because S has VEV and mass of order m_R .

In our model there are symmetries F , T , and \mathbf{z}_α .⁴ As emphasized at the end of the last section, both F and T are broken at the seesaw scale m_R , while the three symmetries \mathbf{z}_α stay valid below m_R and are broken, both softly and spontaneously, only at the electroweak scale m_F . Thus, at the relevant energy scales, *i.e.* in between m_R and m_F , the symmetries \mathbf{z}_α hold. We therefrom conclude that in our model the only non-zero matrices $\kappa^{(ij)}$ are those with $i = j$.

Upon spontaneous symmetry breaking the light-neutrino mass terms are

$$\frac{1}{2} \sum_{\alpha, \beta} (\mathcal{M}_\nu)_{\alpha\beta} \nu_{L\alpha}^T C^{-1} \nu_{L\beta} \quad \text{with} \quad \frac{\mathcal{M}_\nu}{2} = \sum_{i,j} v_i v_j \kappa^{(ij)}, \quad (27)$$

where v_i is the VEV of ϕ_i^0 . Since only the $\kappa^{(ii)}$ are non-zero, we have, at any energy scale,

$$\mathcal{M}_\nu = 2 \sum_i v_i^2 \kappa^{(ii)}. \quad (28)$$

The matrices $\kappa^{(ii)}$ are symmetric, as is obvious from equation (26).

At the high scale m_R , only the Higgs doublet ϕ_0 (and also the scalar singlet S) has Yukawa couplings to the right-handed neutrinos, while the other three Higgs doublets ϕ_α have no such couplings. We therefrom conclude that, at the scale m_R ,

$$\kappa^{(00)} = \frac{\mathcal{M}_\nu}{2v_0^2}, \quad \kappa^{(ii)} = 0_{3 \times 3} \text{ for } i = e, \mu, \tau. \quad (29)$$

This is the initial condition for the renormalization-group (RG) running, with \mathcal{M}_ν given by equation (1).

Let us write the scalar potential of the Higgs doublets in the form

$$V = \text{quadratic terms} + \sum_{i,j,k,l} \lambda_{ijkl} \phi_i^\dagger \phi_j \phi_k^\dagger \phi_l. \quad (30)$$

⁴There is also the CP symmetry. The reasoning and the conclusions in this section are, however, independent of the presence or absence of that CP symmetry. Thus, the present section and its conclusions apply as well to the model of [3].

Let us furthermore write the Yukawa couplings of the Higgs doublets to the right-handed charged leptons in the form

$$\mathcal{L}_{Y\ell} = - \sum_i \sum_{\alpha,\beta} (Y_i)_{\alpha\beta} \bar{D}_{\alpha L} \beta_R \phi_i + \text{H.c.} \quad (31)$$

Then, the RG equations for the evolution of the matrices $\kappa^{(ii)}$ are [12, 13]

$$\begin{aligned} 16\pi^2 \frac{d\kappa^{(ii)}}{dt} = & (-3g^2 + 2T_{ii}) \kappa^{(ii)} + \kappa^{(ii)} P + P^T \kappa^{(ii)} - 2 \left[\kappa^{(ii)} Y_i Y_i^\dagger + Y_i^* Y_i^T \kappa^{(ii)} \right] \\ & + 4 \sum_j \lambda_{jij} \kappa^{(jj)}. \end{aligned} \quad (32)$$

Here, $t = \ln \mu$ is the logarithm of the mass scale, g is the $SU(2)$ gauge coupling constant, and

$$T_{jk} = \text{tr} \left(Y_j^\dagger Y_k \right), \quad (33)$$

$$P = \frac{1}{2} \sum_j Y_j Y_j^\dagger. \quad (34)$$

If we define the 3×3 matrices

$$P_0 = 0_{3 \times 3}, \quad P_e = \text{diag}(1, 0, 0), \quad P_\mu = \text{diag}(0, 1, 0), \quad P_\tau = \text{diag}(0, 0, 1), \quad (35)$$

then we see in equation (15) that, in our model, at the scale m_R ,

$$Y_i = y_1 P_i \quad \text{for } i = 0, e, \mu, \tau, \quad (36)$$

and, therefore,

$$T_{ee} = T_{\mu\mu} = T_{\tau\tau} = |y_1|^2 \text{ and all other } T_{ij} = 0, \quad P = \frac{|y_1|^2}{2} \mathbb{1}_{3 \times 3}. \quad (37)$$

Now one can easily check that the matrices Y_i remain of the form (36) at all energies when they evolve with the RG equations for the Yukawa couplings. We therefore have

$$16\pi^2 \frac{d\kappa^{(00)}}{dt} = (-3g^2 + |y_1|^2) \kappa^{(00)} + 4 \sum_j \lambda_{j0j0} \kappa^{(jj)}, \quad (38)$$

$$16\pi^2 \frac{d\kappa^{(\alpha\alpha)}}{dt} = (-3g^2 + 3|y_1|^2) \kappa^{(\alpha\alpha)} - 2|y_1|^2 \{ \kappa^{(\alpha\alpha)}, P_\alpha \} + 4 \sum_j \lambda_{j\alpha j\alpha} \kappa^{(jj)}, \quad (39)$$

where the anti-commutator of matrices A and B is denoted $\{A, B\}$.

We observe that the coefficients λ_{jij} are particularly important in equations (38) and (39). According to [12], the RG equation for those coefficients is

$$\begin{aligned} 16\pi^2 \frac{d\lambda_{jij}}{dt} = & 4 \sum_{k,l} (2\lambda_{ijkl} \lambda_{lkij} + \lambda_{ijkl} \lambda_{iklj} + \lambda_{iklj} \lambda_{klji} + \lambda_{ikil} \lambda_{kjlj} + \lambda_{kjil} \lambda_{iklj}) \\ & - \left(9g^2 + 3g'^2 - 2T_{ii} - 2T_{jj} \right) \lambda_{jij} \end{aligned} \quad (40)$$

whenever $i \neq j$. In equation (40), g' is the $U(1)$ gauge coupling constant. A little contemplation of equation (40) allows one to ascertain that, if all the λ_{ijij} with $i \neq j$ vanish at the scale m_R , then no such non-vanishing coefficient will ever arise through the RG evolution in our model.

Now, this is precisely the situation that occurs. As seen in equation (20), at the scale m_R , *i.e.* at the initial condition for the RG evolution, there is no term $\left(\phi_i^\dagger \phi_j\right)^2$ with $i \neq j$ in the scalar potential. Therefore, all coefficients λ_{ijij} with $i \neq j$ vanish at the scale $\mu = m_R$, and, through equation (40), at all other scales too.

Equations (38) and (39), together with the initial condition (29), then allow us to ascertain that, in our model, *all matrices $\kappa^{(ij)}$ except $\kappa^{(00)}$ vanish at all energy scales*. The sole non-vanishing $\kappa^{(00)}$ matrix evolves according to

$$16\pi^2 \frac{d\kappa^{(00)}}{dt} = (-3g^2 + |y_1|^2 + 4\lambda_{0000}) \kappa^{(00)}, \quad (41)$$

and therefore it does not change its form along the RG evolution. This means that *all the predictions of our model are RG-invariant*.

In summary this result comes from the fact that the symmetries of our model suitably constrain both the Yukawa and the quartic Higgs couplings, and that soft and spontaneous breaking at the seesaw scale—which is necessary in our model for obtaining the desired \mathcal{M}_ν —has no impact on their RG equations.

4 The effective mass in neutrinoless $\beta\beta$ decay

In our model the effective mass relevant for neutrinoless $\beta\beta$ decay is

$$m_{\beta\beta} = |(\mathcal{M}_\nu)_{ee}| = |x + y|. \quad (42)$$

In order to compute this we have to diagonalize \mathcal{M}_ν . We define [4]

$$U_{\text{HPS}} \equiv \begin{pmatrix} 2/\sqrt{6} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \quad (43)$$

and compute

$$U_{\text{HPS}}^T \mathcal{M}_\nu U_{\text{HPS}} = \begin{pmatrix} p+q & 0 & iq \\ 0 & x+2z & 0 \\ iq & 0 & p-q \end{pmatrix} \quad \text{with } p = x - z \text{ and } q = \frac{3y}{2}. \quad (44)$$

Therefore, defining

$$V = U_{\text{HPS}} D_i K, \quad \text{with } D_i = \text{diag}(1, 1, i) \text{ and } K = \begin{pmatrix} \cos \alpha & 0 & \sin \alpha \\ 0 & 1 & 0 \\ -\sin \alpha & 0 & \cos \alpha \end{pmatrix}, \quad (45)$$

it is evident that the mass matrix \mathcal{M}_ν can be diagonalized as

$$V^T \mathcal{M}_\nu V = \text{diag}(\lambda_1, \lambda_2, \lambda_3), \quad (46)$$

where the neutrino masses are given by $m_j = |\lambda_j|$ ($j = 1, 2, 3$) and the full diagonalization matrix is

$$U = V \text{diag}(\eta_1, \eta_2, \eta_3), \quad (47)$$

where $\eta_j = 1$ for $\lambda_j > 0$ and $\eta_j = i$ for $\lambda_j < 0$.

Then,

$$\begin{aligned} (\mathcal{M}_\nu)_{ee} &= \lambda_1 (V_{e1}^*)^2 + \lambda_2 (V_{e2}^*)^2 + \lambda_3 (V_{e3}^*)^2 \\ &= \frac{2\lambda_1 + \lambda_2}{3} + (\lambda_3 - \lambda_1) \frac{2\sin^2 \alpha}{3} \\ &= \frac{2\lambda_1 + \lambda_2}{3} + (\lambda_3 - \lambda_1) \frac{1 - \cos 2\alpha}{3} \\ &= \frac{\lambda_1 + \lambda_2 + \lambda_3}{3} + \frac{(\lambda_1 - \lambda_3) \cos 2\alpha}{3}. \end{aligned} \quad (48)$$

Since

$$\text{diag}(\lambda_1, \lambda_3) = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} p+q & -q \\ -q & -p+q \end{pmatrix} \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix}, \quad (49)$$

one finds that

$$\sin 2\alpha = \frac{-q}{\epsilon \sqrt{p^2 + q^2}}, \quad (50)$$

$$\cos 2\alpha = \frac{-p}{\epsilon \sqrt{p^2 + q^2}}, \quad (51)$$

$$\lambda_1 = q - \epsilon \sqrt{p^2 + q^2}, \quad (52)$$

$$\lambda_3 = q + \epsilon \sqrt{p^2 + q^2}, \quad (53)$$

where $\epsilon = \pm 1$. Thus,

$$\lambda_1 \lambda_3 = -p^2 < 0, \quad (54)$$

hence

$$p = \eta \sqrt{-\lambda_1 \lambda_3}, \quad (55)$$

where $\eta = \pm 1$. From equations (51)–(53), we obtain

$$\cos 2\alpha = \frac{-2p}{\lambda_3 - \lambda_1}. \quad (56)$$

Returning to equation (48), one obtains

$$(\mathcal{M}_\nu)_{ee} = \frac{\lambda_1 + \lambda_2 + \lambda_3 + 2\eta \sqrt{-\lambda_1 \lambda_3}}{3}. \quad (57)$$

Now we must examine the matter of sign ambiguities in $(\mathcal{M}_\nu)_{ee}$. The three λ_j may be either positive or negative, but they are subject to the condition $\lambda_1\lambda_3 < 0$. Thus,

$$\lambda_1 = \zeta m_1, \lambda_2 = \varepsilon m_2, \lambda_3 = -\zeta m_3, \quad \text{with } \zeta = \pm 1 \text{ and } \varepsilon = \pm 1. \quad (58)$$

On the other hand,

$$|U_{e3}|^2 = \frac{2 \sin^2 \alpha}{3} \quad (59)$$

is experimentally known to be very small, or even zero. Hence,

$$\cos 2\alpha = 1 - 3|U_{e3}|^2 > 0. \quad (60)$$

Therefore, from equations (55) and (56),

$$\frac{-2\eta\sqrt{-\lambda_1\lambda_3}}{\lambda_3 - \lambda_1} > 0, \quad (61)$$

or

$$\frac{2\eta\sqrt{-\lambda_1\lambda_3}}{\zeta(m_1 + m_3)} > 0. \quad (62)$$

Therefore,

$$\eta\sqrt{-\lambda_1\lambda_3} = \zeta\sqrt{m_1m_3}. \quad (63)$$

Thus, from equation (57),

$$3(\mathcal{M}_\nu)_{ee} = \varepsilon m_2 + \zeta(m_1 - m_3 + 2\sqrt{m_1m_3}). \quad (64)$$

There are therefore two possibilities:

$$\begin{aligned} \text{either } m_{\beta\beta} &= \frac{1}{3} |m_2 + m_1 - m_3 + 2\sqrt{m_1m_3}|, \\ \text{or } m_{\beta\beta} &= \frac{1}{3} |m_2 - m_1 + m_3 - 2\sqrt{m_1m_3}|. \end{aligned} \quad (65)$$

Both these possibilities exist independently of whether the neutrino mass spectrum is normal or inverted. Thus, besides (65),

$$\begin{aligned} \text{either } m_1 &= m_s, \quad m_2 = \sqrt{m_s^2 + \Delta m_\odot^2}, \quad m_3 = \sqrt{m_s^2 + \Delta m_{\text{atm}}^2}, \\ \text{or } m_1 &= \sqrt{m_s^2 + \Delta m_{\text{atm}}^2}, \quad m_2 = \sqrt{m_s^2 + \Delta m_{\text{atm}}^2 + \Delta m_\odot^2}, \quad m_3 = m_s. \end{aligned} \quad (66)$$

In figure 1 we have plotted $m_{\beta\beta}$ as a function of s_{13}^2 , taking into account all four possibilities explained above. The input values for the mass-squared differences are $\Delta m_{\text{atm}}^2 = 2.40 \times 10^{-3} \text{ eV}^2$ and $\Delta m_\odot^2 = 7.65 \times 10^{-5} \text{ eV}^2$, the mean values given in the second paper of [2].

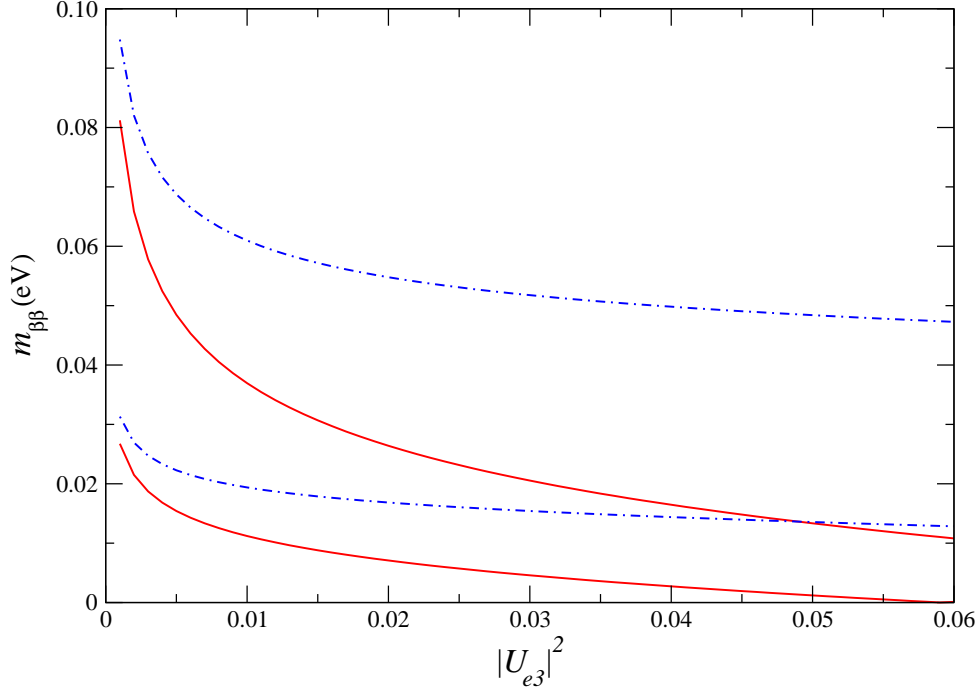


Figure 1: The effective mass in neutrinoless $\beta\beta$ decay, $m_{\beta\beta}$, as a function of $|U_{e3}|^2$. The full lines and the dashed-dotted lines correspond to a normal and an inverted neutrino mass spectrum, respectively. We have fixed the neutrino mass-squared differences at their mean values given in the second paper of [2].

5 Conclusions

We have presented in this paper a model whose symmetries force the charged-lepton mass matrix to be diagonal while generating the highly predictive three-parameter neutrino mass matrix of (1). It follows from this neutrino mass matrix that the lepton mixing matrix U has only one free parameter, which may be chosen to be s_{13} , while both atmospheric-neutrino mixing and CP violation are fixed and maximal. Note that, while U displays a maximal Dirac phase, it has vanishing Majorana phases. Since the model leads to trimaximal mixing, s_{12}^2 must be larger than $1/3$ —see equations (6) and (7). This is slightly disfavoured by the present data, but the value $1/3$ is still within the 2σ range for s_{12}^2 . In any case, the correlation (6) between s_{12}^2 and s_{13}^2 is a crucial test of trimaximal mixing.

In our model, the symmetry which leads to maximal atmospheric-neutrino mixing and maximal CP violation is the non-standard CP transformation given by equations (13) and (14). A CP symmetry of this type has the curious property that $m_\mu \neq m_\tau$ is an effect of its spontaneous breaking, as was noticed earlier in [5, 14].

Given s_{13}^2 and Δm_{atm}^2 , the mass matrix (1) fixes the absolute neutrino mass scale—see equation (8)—but does not determine the type of neutrino mass spectrum. Because of a sign ambiguity there are, for each type of spectrum, two possibilities for the effective mass $m_{\beta\beta}$ of neutrinoless $\beta\beta$ decay as a function of s_{13}^2 . If the neutrino mass spectrum

is of the inverted type, one of the possibilities for $m_{\beta\beta}$ is within the projected range of future experiments.

We have also presented in this paper a mechanism for the one-loop renormalization-group stability of the neutrino mass matrix. Indeed, although some soft and spontaneous breaking of symmetries occurs already at the seesaw scale, the symmetries of our model are such that, in between the seesaw and the electroweak energy scales, only the effective dimension-5 neutrino-mass operator associated with the Higgs doublet ϕ_0 is non-zero, but that Higgs doublet is different from the Higgs doublets $\phi_{e,\mu,\tau}$ which give mass to the charged leptons. Therefore, the form of the neutrino mass matrix is not RG-distorted by the fact that all three charged leptons have different masses.

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